Spline representations of maximal smoothness on general triangulations

Hendrik Speleers

University of Rome Tor Vergata
speleers@mat.uniroma2.it

Splines on triangulations have widespread applications in many areas, ranging from finite element analysis to computer graphics. Highly smooth spline spaces are often preferred.

When dealing with a general triangulation, to obtain splines of high smoothness in a stable manner, sufficiently large degrees have to be considered. An alternative is to use lower-degree macro-elements that subdivide each triangle into a number of subtriangles (or more general subdomains).

Simplex splines are one of the most elegant generalizations of univariate B-splines to the multivariate setting. They enjoy a nice geometric interpretation and several properties such as smoothness and recursion, knot insertion and degree elevation formulas.

In this talk, after reviewing the main issues concerning the construction of highly smooth splines on triangulations, we consider a family of macro-elements of degree p and maximal smoothness $p-1$ and we discuss the construction of a suitable local representation for the related spline space in terms of simplex splines. In particular, we detail the important cases of C^2 cubic [1], and C^3 quartic [2] macro-elements and we discuss several properties, such as local support, linear independence, and nonnegative partition of unity of the provided simplex spline basis.

The talk is based on joint work with Carla Manni and Tom Lyche.

References

[1] T. Lyche, C. Manni, and H. Speleers. *Construction of C^2 cubic splines on arbitrary triangulations*. Foundations of Computational Mathematics 22, 1309–1350, 2022.

[2] T. Lyche, C. Manni, and H. Speleers. *A local simplex spline basis for C^3 quartic splines on arbitrary triangulations*. Applied Mathematics and Computation 462, 128330, 2024.